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The Implementation of the Concrete-Pictorial-Abstract (C-P-A) Instructional Model in Teaching Primary Mathematics in Thimphu Dzongkhag

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Abstract
This study examined the implementation of the Concrete-Pictorial-Abstract (C-P-A) instructional model in Thimphu district. This is because, the District Education Office had adopted the C-P-A model to improve students' mathematical conceptual understanding. A three-day pedagogy workshop for teachers to address persistent underperformance in primary mathematics was also conducted. A concurrent mixed-method design involving a survey of 271 students and 36 teachers, and teachers' interviews was employed. Some of the key findings revealed that teachers believed in the concrete mode of representation using physical manipulative, whereas students preferred the abstract mode of representation using symbols, words and letters. It was found that the C-P-A instructional model had certain constraints in classroom practice, including challenges related to time, concrete materials, and class size. However, teachers were optimistic that the C-P-A instructional model could enhance students' understanding of mathematics. Regular implementation of the C-P-A model, supported by adequate resources and teacher training could significantly improve mathematical conceptual skills and understanding.

Keywords: Concrete model, Virtual model, Pictorial model, Abstract model, Instructional model

Background

Primary mathematics hold a unique and crucial position in education. It fosters real-life learning and serves as critical juncture for establishing a strong foundation for students. However, when it comes to students' learning and their performance in primary mathematics in Bhutan, there are some genuine concerns. It is clearly shown by the nationwide competency-based assessment for class six conducted in 2017. Similarly, the record from the District Education Office [DEO] (2019, pp.24-25) indicates that average district mean mark of mathematics for three years (2017, 2018 and 2019) was 57.3. The record reveals that only four schools under Thimphu district achieved the average mathematics mean mark while the other six schools did not. The academic performance in Classes IV-VI mathematics for these three years remained stagnant and fell below the district's average mean benchmark and confirmed the need to improve mathematics performance for classes IV to VI.

In line with this finding, the DEO (2021) recommended the need for primary teachers to enhance pedagogical knowledge of using the C-P-A instructional model to impart relevant mathematics education to primary school students. To address this crucial need and equip teachers with effective instructional strategies, a 3-day C-P-A pedagogy training was provided in 2020. However, teachers failed to implement the new strategy due to COVID-19 closure of schools. To remediate this gap, the teachers were trained again in 2022-2023 so that they actively use the C-P-A instructional model in mathematics classrooms.

The C-P-A instructional model is grounded in Bruner's enactive, iconic and symbolic mode of representation to help students gain deeper understanding of mathematical concepts. Furthermore, it is considered effective for students of all backgrounds when incorporating concrete manipulatives, visual aids, and abstract representations (Putri et al, 2018). However, inconsistent implementations of the C-P-A instructional model in schools prompted the researcher to explore the perception of teachers and students.

Research Questions

The study was guided by the following research questions:

i. What is the teacher's perception on implementing the C-P-A instructional model in teaching primary mathematics in Thimphu district?
ii. What is the student's perception of the teacher's implementation of the C-P- A model instructional model in teaching primary mathematics?
iii. What are the challenges faced by teachers while implementing the instructional model?
iv. What are the specific scopes of teaching and learning that teachers and students should consider when implementing the C-P-A instructional model in mathematics classrooms?

Literature Review

The Origin of the C-P-A Model

The C-P-A instructional model for teaching mathematics was first proposed in 1966 by an American psychologist Jerome Bruner. Bruner believed that students learn best when building on their existing knowledge and experience. The C-P-A instructional model had three essential interlinked instructional steps known as 'the Concrete', 'the Pictorial', and 'the Abstract'. It was Bruner's three modes of learning – enactive, iconic, and symbolic modes – that had become influential for learning progression (Leong et al., 2015).

Later in the early 1980s, Bruner's three modes of learning were used as the key instructional strategy for the development of primary mathematics concepts in Singapore (Putri et al., 2020). According to Wen (2018, p. 234), Bruner recommended cognitive structure as representation and divided it into three types: 'action representation', 'image representation', and 'symbolic representation'. Since then, Singapore's mathematics education adopted the C-P-A instructional model and based curriculum practices on Bruner's Theory of Learning (Putri et al., 2018).

Concepts and Definitions

Putri et al. (2018) defined the C-P-A instructional model as the learning activates with experiences from actions undertaken (enactive). It was subsequently interpreted into images of the experience formed (iconic) which guided a student to make meaning in abstract concepts. The C-P-A instructional model was also a teaching approach that used concrete objects, pictorial illustrations, and abstract symbols to help students learn mathematics. It was based on the theory that students learned best when they could see and manipulate concrete objects (Hurrell, 2018).

As students developed their understanding of mathematical concepts, they could gradually move to pictorial representations and then to abstract symbols. Therefore, the C-P-A instructional model was an effective strategy for improving students' achievement in mathematics (Rayner, 2016; Sarama & Clements, 2009). According to Rayner (2016, para.2-4), the three-step C-P-A instructional model was highly effective in teaching mathematical concepts.

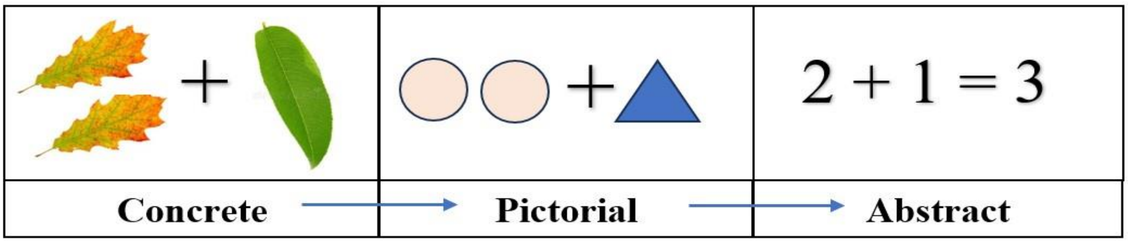
> ***Concrete Mode of Representation****: The 'doing' stage – Students use concrete objects, such as blocks, counters, or manipulatives, to model mathematical concepts by physically moving objects to discover a concept. This helps bring mathematics to connect real-life applications.*

Pictorial Mode of Representation*: The 'seeing' stage – Students use pictures, diagrams, or models to represent mathematical concepts. The stage inspires children to make a conceptual construction between the physical object they just touched and the abstract concept through diagrams and pictures.*

Abstract Mode of Representation*: The 'abstract' stage – Students use abstract symbols, such as numbers and letters, to epitomize mathematical concepts. Teacher uses operation symbols (+, –, x, ÷) to indicate addition, subtraction, multiplication, or division.*

Figure 1

Implementing the C-P-A Sequence in Teaching and Learning



Source adapted: *Hurrell (2018); Leong et al (2015); Purwadi et al (2019); Putri et al (2015); Rayner (2016)*

As shown in Figure 1, the C-P-A instructional model is a way to teach mathematics that starts with using real objects (i.e. concrete form), then moves to pictures (i.e. iconic), and finally ends with words, numbers, and symbols (i.e. abstract form). This helps students understand mathematics better and solve problems more easily. It bridges the gap between abstract concepts and real-world experiences through real objects, images, and symbols thereby effectively facilitating teachers' work and encouraging students' engagement in mathematics (Milton et al., 2019; Mudaly & Naidoo, 2015). Students first capture ideas through concrete manipulatives (Hurrell, 2018) and then create pictures that solidify the connection between the physical and the symbolic (Arcavi, 2003; Mudaly & Naidoo, 2015; Robert, 2019). The sequence is flexible, ensuring that all three phases are connected to create a solid mathematical foundation (Hoe & Jeremy, 2014).

Theoretical Lenses

In traditional mathematics education, students often struggled to connect abstract concepts to real-world experiences. The C-P-A instructional model addressed these issues by providing a step-by-step approach to building understanding of mathematical concepts and problem-solving through concrete objects, images, and finally abstract symbols. Studies found that the C-P-A instructional model improved mathematical skills and deepened conceptual understanding in students of all ages (Hui et al., 2017; Purwadi et al., 2019; Putri et al., 2018). This method contrasted with traditional lecture-based approaches that had minimal impact on learning (Johnson, 2017). The C-P-A instructional model, on the contrary, promoted engagement through hands-on activities and interactive learning (Yulia & Putri, 2021). Previous studies on the Representational Models (RM) helped students interpret real-world situations but were not effective for all learners (Debrenti, 2013). However, the C-P-A instructional model helped in incorporating both external representations (concrete objects and pictures) and internal representations (students' mental connections between concepts).

Additionally, Supandi et al. (2018) highlighted the importance of visual and non-visual learning styles, and the C-P-A instructional model responded to that through the use of concrete materials and manipulatives. Similarly, the Enactive-Iconic-Symbolic-Problem-Based Learning (EISPBL) model aimed to develop problem-solving skills but proved difficult to implement for both teachers and students due to its complexity (Masula & Fauzan, 2019). Moreover, the Concrete-Representational-Abstract (C-R-A) model was similar to the C-P-A instructional model and emphasized the value of manipulatives in learning (Hurrell, 2018). The past study of Paul and Hlanganipai (2014) further suggested that this approach was effective for older students in understanding mathematical concepts.

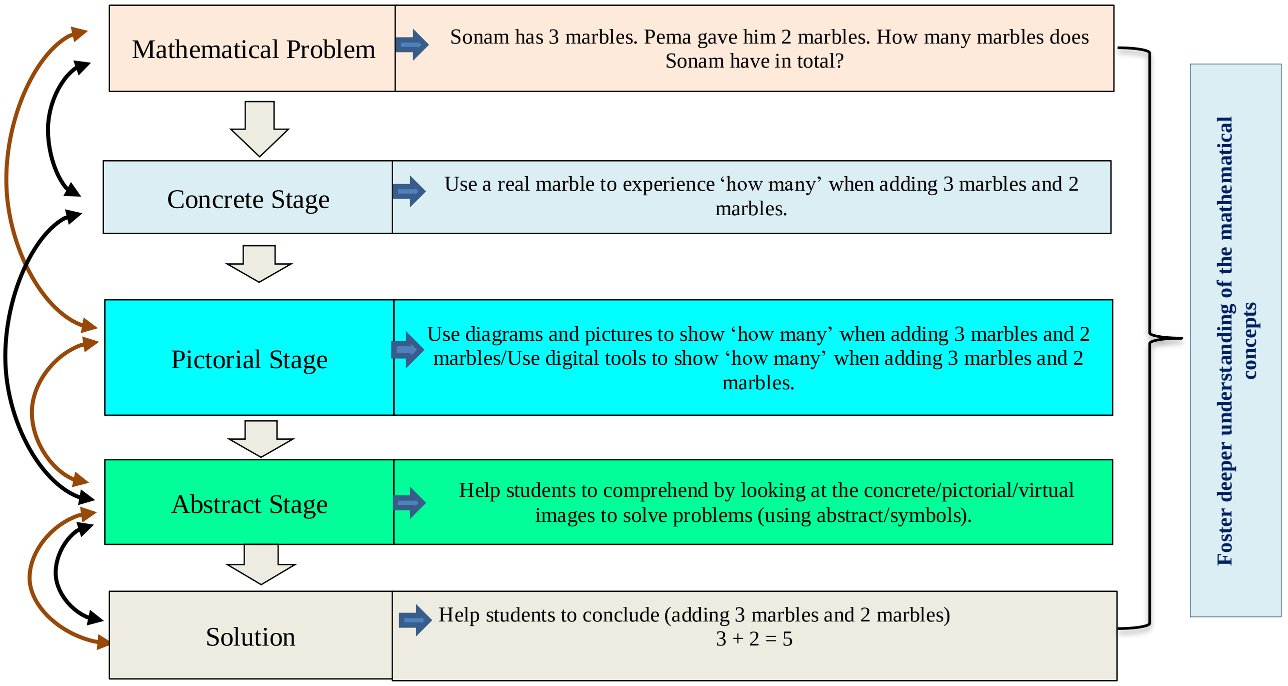
Consequently, Hoe and Jeremy (2014) suggested that advances in technology had led to the C-V-P-A model, which incorporated virtual manipulatives along with concrete and pictorial objects. Virtual manipulatives, such as simulations and animations, could be particularly motivating for students. This approach allowed teachers the flexibility to use physical objects, virtual presentations, or both depending on teaching and learning situations. The versatility of the C-V-P-A model extended beyond mathematics education to science, social studies, and language arts applications (Alper et al., 2017; Hoe and Jeremy, 2014). It further proved to be effective in helping students understand abstract concepts in a variety of subjects (Hoe & Jeremy, 2014; Purwadi et al., 2019). Technology further enhanced the C-P-A instructional model by allowing students to explore and connect with virtual representations (Dockendorff & Solar, 2018).

Practical Application of C-P-A instructional model in the classroom

The C-P-A instructional model benefited both teachers and students (Debrenti, 2013). Teachers could use different representations such as concrete objects, pictures, and abstract symbols to promote student understanding. This approach also allowed teachers to assess student understanding in different ways (Selvianiresa & Prabawanto, 2021). Ultimately, the C-P-A instructional model taught students to enhance problem-solving skills and develop a deeper understanding of mathematical concepts (Mudaly & Naidoo, 2015). It promoted positive attitude towards learning and improved retention through hands-on activities (Arcavi, 2003; Choden & Chalermnirundorn, 2021). The C-P-A instructional model was suitable for a range of mathematical concepts, from basic arithmetic to geometry (Robert, 2019). In particular, the concrete-pictorial-abstract sequence, including virtual manipulatives, was flexible, (Hoe & Jeremy, 2014).

Figure 2

The Application of C-P-A instructional model in classroom instruction



Source: *Hurrell (2018); Leong et al (2015); Purwadi et al (2019); Putri et al (2015); Rayner (2016)*

The application of the C-P-A instructional model in teaching mathematics is depicted in Figure 2. It shows how to use concrete manipulatives to introduce a mathematical concept (e.g. addition), then move to a pictorial mode of representation and to an abstract mode of representation, gaining a deeper understanding of the mathematical concepts. This incremental method helps teachers to make abstract concepts easier for students.

Salingay and Tan (2018) supported that the C-P-A instructional model would help students to exhibit a positive outlook on their mathematical abilities, recognise the significance of mathematics in their daily lives, and demonstrate confidence in their capacity to further develop mathematical knowledge. Similarly, it would help students learn complicated mathematical ideas by interacting with concrete manipulatives to construct meanings and connect this learning experience with the pictorial and abstract mode of representations (Seto et al., 2020).

Hence, the C-P-A instructional model tends to be effective in improving students' attitudes toward learning mathematics, broadening their understanding of mathematical concepts, and boosting their confidence in their abilities (Agustin, 2023). Therefore, students can build a strong foundation in mathematics and apply it in real-world situations by integrating concrete objects, visual aids, and numerical representations.

Methodology

This study employed a concurrent mixed-method design. A sample of 271 students and 36 teachers participated in the survey. In addition, eight teachers were also interviewed. Student participants were selected using proportionate stratified random sampling and teachers were included based on their current teaching subjects in the schools. Data were collected through a validated research instrument, and ethical approval was obtained from all the relevant stakeholders.

Research Paradigm and Design

The study adopted a pragmatist research paradigm, aligning with the goal of investigating the practical implementation of the C-P-A instructional model in primary mathematics classrooms in Thimphu district. A concurrent mixed-methods design was employed to triangulate data from both quantitative and qualitative sources. A close-ended survey was administered to collect quantitative data. To “encourage in-depth responses and greater honesty” (Cohen et al., 2018, p. 508), semi-structured interviews were also conducted with some selected teachers.

Sampling

The sample population comprised of 917 students from grades IV – VI from Thimphu’s nine primary schools. While all 36 primary mathematics teachers were included in the survey, one teacher from each school totalling to eight, were randomly selected for the semi-structured interviews.

Table 1

The Population of Students for the Study

Schools	IV	V	VI	Total
SC01	20	31	29	80
SC02	14	37	25	76
SC03	25	24	19	68
SC04	71	87	67	225
SC05	49	65	49	163
SC06	22	30	32	84
SC07	4	8	5	17

SC08	34	71	35	140
SC09	24	27	13	64
			Total	917

Source: *ADERO V: District Education Office (2022)*

Table 1 indicates the distribution of student population and pseudonyms used in the study. For instance, School 1 is represented as SC01, School 2 as SC02, School 3 as SC03 and so on as shown in Table 1. The students sample size for the quantitative method was 271 which was computed by using Cochran's formula ($n = \frac{Nz^2pq}{N(e)^2+z^2pq}$), where N is the targeted total population (917), with precision level (e= 5% or 0.05). The probability (p) considered for the sampling (50%=0.5), with the cumulative probability of (97.5% = 0.975) determines the value for z (1.96).

Table 2

Sample of Students Participants for the Study

Stratum 1 (School)			**Stratum 2 (Classes IV-VI)**			**Final Sample (Proportionate 2)**		
School	Total	Proportionate 1	IV	V	VI	IV	V	VI
SC01	80	24	20	31	29	6	9	9
SC02	76	23	14	37	25	4	11	8
SC03	68	20	25	24	19	7	7	6
SC04	225	66	71	87	67	21	26	20
SC05	163	48	49	65	49	14	19	14
SC06	84	25	22	30	32	7	9	10
SC07	17	5	4	8	5	1	2	1
SC08	140	41	34	71	35	10	21	10
SC09	64	19	24	27	13	7	8	4
Total	**N= 917**	**n=271**	263	380	274	77	112	82
			917			**271**		

Source: *ADERO V: Dzongkhag Education Office (2022)*

Consequently, as shown in Table 2, 'stratum' was used to categorize schools (Stratum 1) and grade levels (Stratum 2) to ensure sample representativeness and precision. The technique was also used to choose student participants in the identified schools so as to "emphasize representativeness of the entire population" (Iliyasu & Etikan, 2021, p. 24).

Participants

Of the 917 eligible students, 271 were selected to participate in the student survey. During the time of the study, there were 36 primary mathematics teachers in Thimphu. All 36 primary mathematics teachers were included in the survey. From the total of 36 mathematics teachers, eight teachers (one from each identified school) were selected for the interview. The interview data presented under pseudonyms (e.g. Teacher 1 as Tr01) were utilized to support the findings and discussions.

Research Instruments

A quantitative survey, comprising 17 closed-ended questions aligned with established primary mathematics concepts and the C-P-A instructional model was administered to 271 students and 36 teachers to assess their perceptions. Additionally, qualitative data was collected through interviews from eight teachers so as to explore challenges and opportunities in implementing the C-P-A instructional model in primary mathematics instruction. The survey instrument referred a seven-point Likert scale to gauge levels of agreement, with scores above 4.5 indicating a positive response (Pimentel, 2019). A seven-point Likert scale was used to assess responses, ranging from Strongly Disagree (1) to Strongly Agree (7), with intermediate points representing nuances of agreement or disagreement.

Data Collection Procedure

The researcher personally visited nine schools under Thimphu district to ensure the efficient and accurate collection of data. To minimize distractions and facilitate focused responses, questionnaires were administered to 271 students and 36 teachers in a quiet room. Subsequently, semi-structured interviews were conducted with eight selected teachers within a week. To maintain objectivity and impartiality, school administrators were asked to identify a group of teachers to conduct student surveys, while the researcher personally administered the teacher surveys and conducted teacher interviews. This approach aimed to ensure a neutral data collection process and maximize the reliability of the findings.

Ethical Consideration

This research ensured the ethical treatment of participants. Permissions were obtained from authorities and informed consent was secured. Participants' comfort, safety, and confidentiality were also prioritized throughout the data collection process (Creswell, 2014).

Analysis

Quantitative data were analyzed using SPSS, employing descriptive statistics, reliability analysis, factor analysis, and inferential statistics to examine central tendencies, variability, instrument reliability, underlying structures, and relationships between variables. Subsequently, the qualitative data were analyzed thematically, involving transcription, coding, and identifying patterns and themes. The integration of these quantitative and qualitative analyses provided a comprehensive understanding of the C-P-A instructional model's implementation.

Reliability and Validity

The study focused on reliability and accuracy. The researcher used established methods such as consistent measurement and pilot testing (surveys and interviews) to capture the intended information. Pilot testing with students and teachers identified areas for improvement. For example, the survey's topic 'concrete mode of representation' lacked clarity, so the item was revised. Similarly, the first question in the teacher interview was adjusted to probe how the C-P-A instructional model was implemented. Overall, these steps enhanced the reliability (consistency) and validity (trustworthiness) of the research methodology. According to Cronbach (1951), Cronbach's alpha assesses a survey's reliability by measuring internal consistency. It checks if the questions target the same concept, with a value of 0.70 or higher considered acceptable (Taber, 2018).

Table 3

Reliability Test

C-P-A Components	Alpha Coefficient	
	Students	Teachers
Concrete Mode of Representation	0.72	0.87
Pictorial Mode of Representation	0.65	0.78
Abstract Mode of Representation	0.71	0.64
Overall	0.69	0.76

As indicated in Table 3, the survey instrument showed good internal consistency and demonstrated reliable measurement. All values of the three components (concrete, pictorial, and abstract) were above 0.64 for both students' and teachers', exceeding Taber's (2018) recommended threshold of 0.70. Notably, both students and teachers had the highest reliability for the concrete mode of representation questions and the lowest reliability for the abstract mode of representation questions. Overall, the Cronbach's alpha values of 0.69 for students and 0.76 for teachers indicate that the survey in the context of this study was reliable.

Factor Analysis

Exploratory factor analysis (EFA) simplifies complex survey data by grouping related questions into factors (Surucu et al., 2022). It checks if the data is suitable for analysis using the Kaiser-Meyer-Olkin test and Bartlett's test (Williams et al., 2010).

Table 4

Data Suitability Test

Kaiser-Meyer-Olkin (MSA)		0.84
Bartlett's Test of Sphericity	Approx. Chi-Square	1553.4
	Df	136
	Sig.	0.00

The result in Table 4 suggested that the data were suitable for exploratory factor analysis (EFA) with a high Kaiser-Meyer-Olkin (KMO) value (0.84) and a significant Bartlett test ($p<0.05$). However, three survey items were later removed because they did not align well with the intended factors. These points described the use of local objects (CM3), visual learning diagrams (PM5), and number line diagrams (PM1). Even after removing these points, the KMO value remained high (0.82) and the Bartlett test remained significant ($p<0.05$), confirming that the remaining 10 points were still suitable for further EFA.

Table 5

Exploratory Factor Analysis (EFA)

	Component		
	1	2	3
CM_1	.77		
CM_2	.83		
CM_4	.64		
PM_2		.56	
PM_3		.67	
PM_4		.85	
AM_1			.77
AM_2			.57
AM_3			.74
AM_4			.66

The result in Table 5 indicated that the identified three factors through EFA test aligned with the Concrete, Pictorial, and Abstract components of the C-P-A instructional model for further analysis.

Findings and Discussions

Based on data analysis (descriptive and inferential), the perceptions on the implementation of the C-P-A instructional model were studied. Teachers were further interviewed to present the comprehensive finding and insights on the effectiveness of the instructional model.

Teacher Survey Result

A 17-item close-ended survey questionnaire was administered to teacher respondents to investigate their perceptions on the implementation of the C-P-A instructional model as well as to evaluate their own implementation of the model. Teacher respondents completed the survey questionnaire utilizing a seven-point Likert scale based on Pimentel (2019). Table 6 presents the mean (M) and standard deviation (SD) of the three components of the C-P-A instructional model as perceived by teachers.

Table 6

Teacher -participants' perceptions on the C-P-A Instructional Model

	N	Mean	Std. Deviation
Concrete Mode of Representation	36	6.47	0.74
Pictorial Mode of Representation	36	6.36	0.54
Abstract Mode of Representation	36	6.36	0.41
Overall (Mean)	36	6.39	0.56

The mean scores of teacher respondents with respect to the C-P-A instructional model components were typical of strong agreement (Concrete Mode of Representation: M = 6.47, SD = 0.74; Pictorial Mode of Representation: M = 6.36, SD = 0.54; Abstract Mode of Representation: M = 6.36, SD = 0.41) with some variations in views. The result is indicative of the fact that teachers have strong agreement of using all the C-P-A components in classroom instructions. According to the result, teachers also strongly believed in using concrete mode of representation in instructions compared to pictorial and abstract mode of representations. This supported the findings of past studies (Debrenti, 2013; Yulia & Putri, 2021) which postulated that students' initially enhance their learning through hands on activities and interactions with world around them. Similarly, one of the teacher interviewees stated that *"children learn better when there are hands-on activities. I think this is one of the best ways for the learners to retain. They can retain for a longer time"* (TR01). This also showed that to build strong foundation in mathematics, students must experience and feel the concepts and handle real world concrete objects.

Student Survey Result

Similarly, a 17-item close-ended survey questionnaire was administered to student respondents to assess their teachers' implementation of the C-P-A instructional model in the classroom.

Table 7

Student-participants' perception on Teachers Implementation of the C-P-A Instructional Model

	N	Mean	Std. Deviation
Concrete Mode of Representation	271	5.84	1.57
Pictorial Mode of Representation	271	5.88	1.38
Abstract Mode of Representation	271	6.37	0.93
Overall (Mean)	271	6.03	1.29

Students' perceptions of the C-P-A instructional model implementation in their classrooms varied more widely than teachers' perceptions. Table 7 indicates that on an average, students agreed that teachers used the C-P-A instructional model (M = 6.03, SD = 1.29) with the highest scores for the Abstract Model (M = 6.37, SD = 0.93) and the lowest scores for the Concrete Model (M = 5.17, SD = 1.80). The result indicated significant difference in perceptions that teachers failed to implement the C-P-A instructional model consistently. Teacher respondents revealed that fitting all three stages (concrete, pictorial, abstract) within limited class time in the context of having insufficient manipulatives for hands-on activities, and classroom management during these active learning phases have challenged the proper implementation of the C-P-A instructional model. Similarly, the past studies (Hui et al., 2017; Leong et al., 2015; Putri et al., 2018) also revealed that teachers experience time constraints in delivering the prescribed curriculum within allotted instructional hours while extensively aligning the C-P-A instructional model in teaching and learning,

Comparison of Student Respondents' and Teacher Respondents' Perceptions

Additionally, comparing the perceptions of student respondents and teacher respondents regarding the C-P-A instructional model is essential. This comparison provides an understanding of the model's effectiveness and identifies potential areas for improvement. By analyzing both the perspectives, teacher respondents can gain insights into the model's alignment with student and teacher needs, assess its impact, and inform future implementation strategies.

Table 8

Mann-Whitney U Test and the Effect of Significance

	Concrete Mode of Representation	Pictorial Mode of Representation	Abstract Mode of Representation
Mann-Whitney U	3884.000	3809.000	3911.500
Wilcoxon W	40740.000	40665.000	4577.500
Z	-2.128	-2.175	-2.062
Asymp. Sig. (2-tailed)	.033	.030	.039
Effect Size (r)	0.12	0.12	0.12

Grouping Variable: Teachers or Students

As shown in Table 8, there is a statistically significant difference between student respondents' and teacher respondents' perceptions of all components of the C-P-A model (concrete, pictorial, and abstract) ($p < 0.05$) (Surucu et al., 2022). However, the effect size ($r = 0.12$) is small, suggesting that the substantial differences between the two groups are small (Kiess & Green, 2019).

Table 9

The Mean Comparison of C-P-A Components between Teacher respondents and Student Respondents

	N	Mean Rank		
		Concrete Mode of Representation	Pictorial Mode of Representation	Abstract Mode of Representation
Teacher	36	181.61	169.13	127.15
Student	271	150.33	151.99	157.57

Although there is a statistically significant difference between teacher respondents' and student respondents' perceptions on the use of C-P-A instructional models ($p < 0.05$), the effect size is small (Kiess & Green, 2019; Surucu et al., 2022). As indicated in Table 9, teacher respondents' rankings suggest that they feel they use concrete mode of representation most frequently (MR 181.61) and pictorial mode of representation less frequently (MR 169.13). Conversely, student respondents have rated their teachers' use of abstract mode of representation most highly (MR 157.57) compared to concrete mode of representation (MR 127.15).

Relationships among the Components of the C-P-A Instructional Model

The Spearman correlation analysis was further employed to investigate the interrelationships among the components of the CPA instructional model. This statistical technique quantifies the strength and direction of the relationships between these components.

Table 10

Correlation among Concrete, Pictorial, and Abstract Instructional Model

		Spearman Correlation Coefficient					
	N	Concrete Mode of Representation		Pictorial Mode of Representation		Abstract Mode of Representation	
		ρ	Sig.	ρ	Sig.	ρ	Sig.
Concrete	307	1.00	-	0.39**	0.00	0.23**	0.00
Pictorial	307	0.39**	0.00	1.00	-	0.36**	0.00
Abstract	307	0.23**	0.00	0.36**	0.00	1.00	-

*** Correlation is significant at the 0.01 level (2-tailed).*

Table 10 revealed there were positive correlations between all three components of the C-P-A instructional model. The strongest correlation existed between the Concrete and Pictorial mode of representation (rho = 0.39), followed by Pictorial and Abstract mode of representation (rho = 0.36). The Concrete and Abstract mode of representations had the weakest positive correlation (rho = 0.23). These findings suggest that teacher respondents who utilize concrete and pictorial mode of representations are more likely to use abstract mode of representations and vice versa. According to Prion and Haerling (2014), this highlights the interrelatedness of the C-P-A instructional model framework, where each stage reinforces the others in primary mathematics education.

The result is based on the Mann Whitney U- test, mean comparison and correlation. A statistically significant difference was found between student respondents' and teacher respondents' perceptions of all components of the C-P-A instructional model (concrete, pictorial, and abstract) ($p < 0.05$) with a small effect size ($r = 0.12$). Similarly, teachers' rankings as shown in Table 9 revealed that teacher respondents' have the highest MR = 181.61 in 'Concrete Mode of Representation' whereas student respondents' have the highest MR=157.57 in the 'Abstract Mode of Representation'. The result indicates that there is a significant difference between student respondents' and teacher respondents' perceptions on the implementation of the C-P-A instructional model.

These findings contradict with the previous study by Leong et al. (2015) and Putri et al. (2018) particularly with regard to the principal purpose of the instructional model as the model of understanding mathematics. One of the interviewees argued that "*… C-P-A is put into practice,*" and students have to "*use the available resources*" to build "*a brighter future for mathematics*" (Tr04). This highlights that teacher respondents generally appreciate the C-P-A instructional model although practical limitations impact its consistent use resulting to differences in the perceptions. For instance, teacher respondents perceived the concrete mode of representation as the best model to teach concepts, while student respondents mostly experienced the use of abstract mode of representations to learn new mathematical concepts.

To overcome the challenges like time constraints, lack of concrete manipulatives, and large class size with limited space, many past studies suggest that the use of virtual manipulatives can be dynamic representations, and it could facilitate students' ability to solve abstract problems in mathematics (Alper et al., 2017; Hoe & Jeremy, 2014). Furthermore, use of technology in promoting conceptual understanding through virtual representation using animations (Dockendorff & Solar, 2018; Hoe & Jeremy, 2014) also play a key role in understanding mathematical concepts. Similarly, many past studies (Hui et al., 2017;

Peltier & Vannest, 2018; Putri et al., 2018) recommend the use of virtual manipulatives to help students learn mathematics.

Although correlations (as shown in Table 10) revealed a weak positive correlation with the highest concrete and abstract at $\rho = 0.39^{**}$, students' understanding of one model does not necessarily predict their understanding of another. This suggest that teachers might need to explicitly link models to abstract concepts to ensure all students benefit. Lessons that teacher handles may be appropriate to incorporate all the mode of representations in teaching. However, it is the role of the teacher to seek suitable representation while implementing it in teaching and learning process. Conversely, the past studies argue that the teacher's role to support students to progress through the stages of learning with the C-P-A instructional model is often overlooked. This shift seems to do away with one of the teacher's most crucial aspects concerning students' learning (Hui et al., 2017; Purwadi et al., 2019; Putri et al., 2020). Similarly, different views between teacher respondents and student respondents on implementing the C-P-A instructional model can negatively impact learning results. Nevertheless, more concrete experience may be needed to improve students' understanding of abstract ideas requiring teachers to seek students' feedback and changing teaching methods. Thus, using hands-on activities and relating lessons to real-life scenarios can greatly improve students' interest and understanding.

Conclusion

This study examined the theoretical background and practical applications of the C-P-A instructional model, a key method in teaching mathematics. By exploring its origins, main parts, and impact on student learning, the researcher has gained valuable insights into its effectiveness. While the C-P-A instructional model offers a structured approach to teaching mathematical concepts, challenges such as teacher training, resource availability, and student engagement need to be addressed for its successful implementation. Future researchers could explore the long-term effects of C-P-A instructional model on student achievement, especially in diverse learning settings. Eventually, when implemented effectively, the C-P-A instructional model has the potential to revolutionize mathematics education, fostering deeper understanding of mathematical concepts and enhancing students' problem-solving skills.

However, the successful implementation of the C-P-A instructional model will depend on the supports of the relevant stakeholders in providing in-depth teacher training programmes and procuring enough resources. Similarly, classroom management techniques, such as setting clear expectations, applying positive reinforcement, and keeping an even flow, are vital in a large and diversified classroom. Moreover, the diversity of learners can greatly benefit from differentiated instruction, alignment of curriculum with the C-P-A instructional model, flexible assessment strategies, collaboration of teachers, and parental involvement.

Limitations

Some of the participating teachers were new to the concept and implementation of the C-P-A instructional model. These teachers received only a few hours of professional development at the school level. Hence, they were not competent enough to express their opinions on the implementation of the C-P-A instructional model. If all the teachers were crystal clear of the C-P-A concepts, the information from the data analysis could have been different. The findings of the study were also limited to the participating students (n=271) from classes IV to VI and primary school teachers (n=36) under Thimphu district only. They are not representative of students from other districts and city schools in Bhutan. The study did not adopt the theoretical and conceptual framework. Instead, the project was viewed and conducted through theoretical lenses with similar ideas in implementing the instructional model. Because of these reasons, this study does not evaluate or recommend the long-term effects of implementing the C-P-A instructional model on student learning outcomes.

Recommendations

Based on the findings and limitations, this study presents the following recommendations:

- There is a gap between teachers' theoretical knowledge and practical application. Professional development or training programmes focused on the effective implementation of all stages (concrete, pictorial, and abstract) would enhance teachers' overall understanding on the C-P-A instructional model and effective implementation in future.
- Teachers face challenges such as limited time, insufficient resources, and large class sizes. In light of the challenges, considering the incorporation of virtual manipulatives such as digital tools, animations and platforms strategically with the C-P-A instructional model can maximize effectiveness in mathematics teaching and learning process.
- Besides, there is a need for relevant stakeholders to develop policies and programmes to ensure all students have access to computer labs with reliable internet connections. Teachers should have competency in the curation of digital content, tools and platforms relevant to mathematics teaching and learning to effectively integrate virtual manipulatives along with the C-P-A instructional model. In the concrete stage, for instance, virtual manipulatives provided on the website of the National Library of Virtual Manipulatives and the interactive geometry tool like GeoGebra would be beneficial since they would make learning fun and worthwhile. In the pictorial stage, supported by digital whiteboards, Microsoft Whiteboard or Google Jamboard, students can simultaneously jot down their thoughts and represent images as they learn together. In the abstract learning stage, online platforms such as Google Classroom and Khan Academy can enhance the learning experience by offering video explanations and practice exercises, making the material easier to understand.
- Additionally, to broaden the applicability of the findings, future researches should involve a larger sample size and a wider geographical scope, encompassing diverse regions and educational contexts. This will enable a more comprehensive understanding of the model's impact on student learning and teacher practices.

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